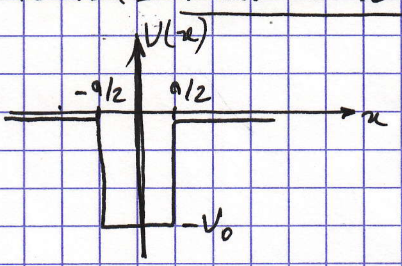
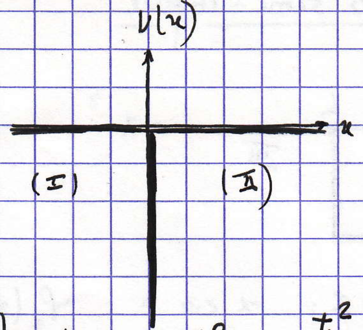
PO F04 _ Puits de Dirac -





a) Eq^o de Schrödinger indépendante du temps : $E\varphi = -\frac{\hbar^2}{2m}\varphi'' - V_0\varphi$

$$\frac{d^2\varphi}{dx^2} = -\frac{2m}{\hbar^2}(E+V_0)\varphi$$

$\times dx$ $$\int_{-a/2}^{a/2}\frac{d^2\varphi}{dx^2}dx = -\frac{2m}{\hbar^2}\int_{-a/2}^{a/2}(E+V_0)\varphi\, dx$$

E fini cst.

$$\varphi'\left(\frac{a}{2}\right) - \varphi'\left(-\frac{a}{2}\right) = -\frac{2m}{\hbar^2}\int_{-a/2}^{a/2}(E+V_0)\varphi\, dx \xrightarrow[a\to 0]{} -\frac{2m}{\hbar^2}(E+V_0)\cdot a\cdot\varphi(0)$$

$\downarrow a\to 0$ $\quad\quad\quad\quad$ $\downarrow$

$$\underline{\varphi'(0^+) - \varphi'(0^-) \simeq -\frac{2m}{\hbar^2}V_0 a\varphi(0)} \longleftarrow -\frac{2m}{\hbar^2}\underbrace{V_0 a}_{\neq 0}\varphi(0)$$

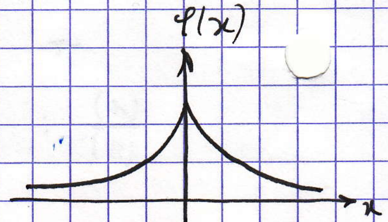
b) $x<0$: $-|E|\varphi = -\frac{\hbar^2}{2m}\varphi'' \longrightarrow \varphi'' - \underbrace{\frac{2m|E|}{\hbar^2}}_{\alpha^2}\varphi = 0$

$$\longrightarrow \varphi_I = A_I e^{\alpha x} + \cancel{B_I e^{-\alpha x}}$$

$\cancel{B_I}$ = 0 car terme divergent.

$x>0$: $\varphi_{II} = \cancel{A_{II}} e^{\alpha x} + B_{II} e^{-\alpha x}$

$\cancel{A_{II}}$ = 0 idem.



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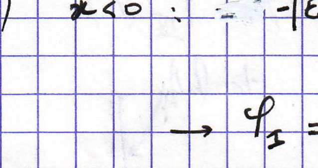
c) Continuité de φ : $\varphi(0) = \underline{A_I = B_{II}} = A$

Discontinuité de φ' : $\varphi'(0^+) - \varphi'(0^-) = -2A\alpha = -\frac{2m}{\hbar^2}aV_0 A$

$$\left.\begin{aligned}\Rightarrow \alpha &= \frac{maV_0}{\hbar^2}\\ \text{or } \alpha &= \frac{\sqrt{2m|E|}}{\hbar}\end{aligned}\right\} \longrightarrow \boxed{E_f = -\frac{ma^2V_0^2}{\hbar^2}}$$

d) Etat lié impair



$$\Rightarrow \begin{aligned}\varphi(x>0) &= Ae^{-\alpha x}\\ \varphi(x<0) &= -Ae^{\alpha x}\end{aligned}$$

$\varphi'(0^+) - \varphi'(0^-) = -A\alpha + A\alpha = 0$ ne vérifie pas la condition limite de discontinuité de φ'