* PO414 - Onde dans un plasma + champ magnétique

1 - * $\underline{\vec{E}} = (E_{0x}\vec{e_x} + E_{0y}\vec{e_y})e^{i(\omega t - kz)}$

* $\underline{\vec{j}} = -n_0 e \underline{\vec{v}}$

* PFD sur $1\bar{e}$: $m\dot{\underline{\vec{v}}} = -e\underline{\vec{E}} - e\underline{\vec{v}} \wedge \vec{B_0}$ dans l'approximation non relativiste.

$$\begin{cases} \dot{\underline{v}}_x = -\frac{e}{m}\underline{E}_x \\ \dot{\underline{v}}_y = -\frac{e}{m}\underline{E}_y - e\underline{v}_z B_0 \\ \dot{\underline{v}}_z = +e v_y B_0 \end{cases} \Rightarrow \begin{cases} \underline{v}_x = +\frac{ie}{m\omega}\underline{E}_x \\ \underline{v}_y = +i\frac{e}{m\omega}\underline{E}_y + i\frac{eB_0}{m\omega}\underline{v}_z \\ \underline{v}_z = -i\frac{eB_0}{m\omega}\underline{v}_y \end{cases}$$

$\times(-en_0) \Rightarrow$

$$\boxed{\begin{cases} \underline{j}_x = -i\frac{e^2 n_0}{m\omega}\underline{E}_x \\ \underline{j}_y = -i\frac{e^2 n_0}{m\omega}\underline{E}_y + i\frac{eB_0}{m\omega}\underline{j}_z \\ \underline{j}_z = -i\frac{eB_0}{m\omega}\underline{j}_y \end{cases}}$$

2 - $\underline{E}_y = 0 \Rightarrow$

$$\begin{cases} j_x = -i\frac{e^2 n_0}{m\omega}E_x \\ j_y = +\frac{ieB_0}{m\omega}j_z \\ j_z = -i\frac{eB_0}{m\omega}j_y \end{cases} \rightarrow \underline{j}_y = \frac{e^2B_0^2}{m^2\omega^2}\underline{j}_y \Rightarrow j_y = 0 \Rightarrow j_z = 0$$

$\Rightarrow \underline{\vec{j}} = \gamma \underline{\vec{E}}$; $\boxed{\gamma = -i\frac{e^2 n_0}{m\omega}}$

Relation de dispersion. $k^2 = \frac{\omega^2}{c^2} - i\mu_0\underline{\gamma}\omega = \frac{\omega^2}{c^2} - \frac{\mu_0 e^2 n_0}{m} = \frac{\omega^2 - \omega_p^2}{c^2}$

où $\boxed{\omega_p^2 = \frac{e^2 n_0}{m\varepsilon}}$

- Propagation si $\omega > \omega_p$
- Onde évanescente si $\omega < \omega_p$
- résonance si $\omega = \omega_p$

3. $E_x = 0 \Rightarrow \begin{cases} j_x = 0 \\ j_y = -\dfrac{e^2 n_0}{m\omega} E_y + i\dfrac{eB_0}{m\omega} j_z \\ j_z = -i\dfrac{eB_0}{m\omega} j_y \end{cases}$

$$\begin{cases} j_x = 0 \\ j_y = -\dfrac{e^2 n_0}{m\omega} E_y + \dfrac{e^2 B_0^2}{m^2\omega^2} j_y \\ j_z = -i\dfrac{eB_0}{m\omega} j_y \end{cases} \qquad \omega_c^2 = \frac{e^2 B_0^2}{m^2}$$

$$\begin{cases} j_x = 0 \\ j_y\left(1 - \dfrac{e^2 B_0^2}{m^2\omega^2}\right) = -\dfrac{e^2 n_0}{m\omega} E_y \\ j_z = -i\dfrac{eB_0}{m\omega} j_y \end{cases} = \begin{cases} j_x = 0 \\ j_y\left(1 - \dfrac{\omega_c^2}{\omega^2}\right) = \dfrac{\varepsilon\omega_p^2}{\omega} E_y \\ j_z = -i\dfrac{\omega_c}{\omega} j_y \end{cases}$$

$$\Rightarrow \boxed{\begin{cases} j_x = 0 \\ j_y = \varepsilon\dfrac{\omega_p^2}{\omega}\dfrac{1}{1-\dfrac{\omega_c^2}{\omega^2}} E_y \\ j_z = -i\varepsilon\dfrac{\omega_p^2\omega_c}{\omega}\dfrac{1}{1-\dfrac{\omega_c^2}{\omega^2}} E_y \end{cases}}$$

$$\vec{j} = [\gamma] E$$

$$\vec{j} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & \beta & 0 \end{pmatrix}\begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix}$$

Eq$^{\circ}$ de propagation $\Delta\vec{E} - \dfrac{1}{c^2}\dfrac{\partial^2\vec{E}}{\partial t^2} = -\mu_0\dfrac{\partial\vec{j}}{\partial t} \rightarrow -k^2\vec{E} + \dfrac{\omega^2}{c^2}\vec{E} = -i\omega\mu_0\vec{j}$

$\cdot\ \vec{e_y}$: $\left(-k^2 + \dfrac{\omega^2}{c^2}\right)E_y = -i\omega\mu_0 j_y = -i\mu_0\varepsilon\omega_p^2 \dfrac{1}{1-\dfrac{\omega_c^2}{\omega^2}} E_y$

$$\boxed{k^2 = \frac{\omega^2}{c^2} + i\frac{\omega_p^2}{c^2}\frac{1}{1-\dfrac{\omega_c^2}{\omega^2}}}$$