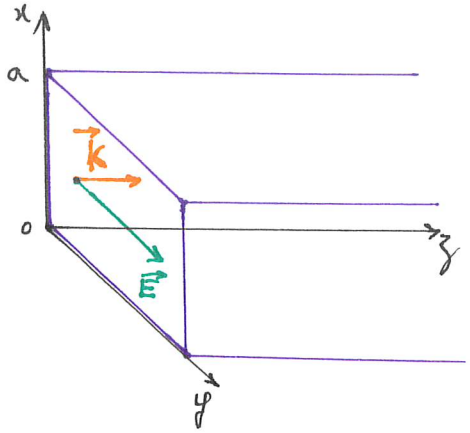
P0318. Electromagnétisme dans un guide d'onde



1- (cf cours) .../...

Équation de propagation :

$$\frac{\partial^2 \vec{E}}{\partial t^2} - c^2 \vec{\Delta} \vec{E} = \vec{0}$$

2- $\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 \vec{E}$

$$\vec{\Delta E} = \left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial z^2}\right) \vec{u}_y$$

$$= -\alpha^2 \vec{E} - K^2 \vec{E}$$

$$\Rightarrow \left(-\frac{\omega^2}{c^2} + \alpha^2 + K^2\right) \vec{E} = \vec{0} \quad \forall\, x, y, t \quad \Rightarrow \quad K^2 = \frac{\omega^2}{c^2} - \alpha^2 \quad (D)$$

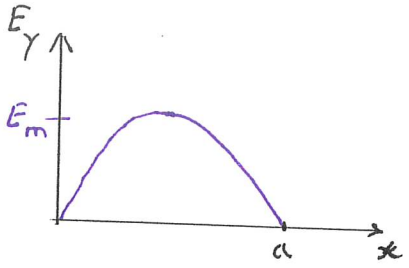
3a) Continuité de E_y en $x = 0$: $E_y(x=0, y, t) = 0$

$x = a$: $E_y(x=a, y, t) = E_m \sin(\alpha a) \cos(\omega t - kz) = 0$

$\Rightarrow \sin(\alpha a) = 0 \rightarrow \alpha a = n\pi$, $n \in \mathbb{N}^*$

$$\alpha_n = n \frac{\pi}{a}$$

b) $\alpha = \frac{\pi}{a}$, $t = 0$, $z = 0$ $\Rightarrow$ $E_y = E_m \sin\left(\frac{\pi x}{a}\right)$



c) K Il y a propagation si $K \in \mathbb{R}$ donc $K^2 > 0$ $\Rightarrow$ $\omega > \alpha c = \omega_c$

Guide d'onde $\Leftrightarrow$ filtre passe-haut.

d) $f_c = \frac{\omega_c}{2\pi} = \frac{\alpha c}{2\pi} = \frac{c}{2a}$ $\quad f_c = 7{,}5\ \text{GHz}$ Micro-onde

4- $v_\varphi = \frac{\omega}{K} = \frac{\omega}{\sqrt{\frac{\omega^2 - \omega_c^2}{c^2}}} \rightarrow v_\varphi = \frac{c}{\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}}$ où $\omega > \omega_c$

$v_g = \frac{d\omega}{dK}$; Différencions (D) : $2K\,dK = \frac{1}{c^2} 2\omega\, d\omega$

$\rightarrow \frac{d\omega}{dK} = c^2 \frac{K}{\omega} = \frac{c^2}{v_\varphi}$ $\Rightarrow$ $v_g = c\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$