Po 214. Réflexion - transmission d'onde sonore sur une membrane

1. Onde incidente $\underline{p}_i = p_0 e^{i(\omega t - k_1 x)}$ $\vec{k}_1 = \frac{\omega}{c_1}\vec{e_x}$

$\underline{v}_i = \frac{p_0}{Z_1} e^{i(\omega t - k_1 x)}$ $Z_1 = \mu_1 c_1$

— réfléchie : $\underline{p}_r = r . p_0 e^{i(\omega t + k_1 x)}$

$\underline{v}_r = -\frac{r\, p_0}{Z_1} e^{i(\omega t + k_1 x)}$

— transmise $\underline{p}_t = t . p_0 e^{i(\omega t - k_2 x)}$ $\vec{k}_2 = \frac{\omega}{c_2}\vec{e_x}$

$\underline{v}_t = \frac{t p_0}{Z_2} e^{i(\omega t - k_2 x)}$

2. Conditions aux limites (en $x = 0$ si membrane ass^{nt} mince et son déplac^{nt} $\ll \lambda$)

* Continuité de la vitesse $\underline{v}_i(0,t) + \underline{v}_r(0,t) = \underline{v}_t(0,t)$

$\frac{1}{Z_1}(1 - r) = \frac{t}{Z_2} \longrightarrow \boxed{1 - r = t}$ (a)

car $Z_1 = Z_2$

* Condition sur la pression : PFD sur membrane :

$\sigma \cancel{S}\, \ddot{x}_{memb} = \left[p_i(0,t) + p_r(0,t)\right] \cancel{S}\vec{e_x} - p_t(0,t) S \vec{e_x}$

où $\underline{v}_m = \underline{v}_i + \underline{v}_r = \underline{v}_t(0,t)$

$i\omega\sigma \underbrace{\underline{v}_t}_{\frac{p_t}{Z_2}} = (p_i + p_r) - p_t \longrightarrow \frac{i\omega\sigma}{Z_1} t = 1 + r - t$

car $Z_1 = Z_2$

$\boxed{t\left(1 + \frac{i\omega\sigma}{Z_1}\right) = 1 + r}$ (b)

$\left(1 + \frac{i\omega\sigma}{Z_1}\right)(a) - (b)$: $\left(1 + \frac{i\omega\sigma}{Z_1}\right)(1 - r) - (1 + r) = 0$

$\frac{i\omega\sigma}{Z_1} - r\left(2 + \frac{i\omega\sigma}{Z_1}\right) = 0 \rightarrow \boxed{r = \frac{i\omega\sigma}{2Z_1 + i\omega\sigma}}$

$(a) + (b)$ $2 = t\left(2 + \frac{i\omega\sigma}{Z_1}\right) \rightarrow \boxed{t = \frac{2Z_1}{2Z_1 + i\omega\sigma}}$

$T = \frac{\langle p_t \cdot v_t \rangle}{\langle p_i \cdot v_i \rangle}$ / $p_i = p_0 \cos(\omega t - k_1 x)$

$v_i = \frac{p_0}{Z_1} \cos(\omega t - k_1 x)$

$p_t = \mathrm{Re}(\underline{t}\, \underline{p}_i) = p_0 \mathrm{Re}(\underline{t}) \times \cos(\omega t - k_2 x) - \mathrm{Im}(\underline{t}) \sin(\omega t - k_2 x)$

$v_t = \frac{1}{Z_2}\mathrm{Re}(\underline{t}\, \underline{p}_i) = \frac{p_0^2}{Z_2}\left(\mathrm{Re}^2(\underline{t}) \cos^2(\,) + \mathrm{Im}^2(\underline{t}) \sin(\,)\right.$

$\left. - 2\mathrm{Re}(\underline{t})\, \mathrm{Im}(\underline{t}) \cos(\,) \sin(\,)\right)$

$\rightarrow \langle p_t v_t \rangle = \frac{p_0^2}{Z_2} \frac{\mathrm{Re}^2(\underline{t}) + \mathrm{Im}^2(\underline{t})}{2} = \frac{1}{2}\frac{p_0^2}{Z_2}|t|^2$

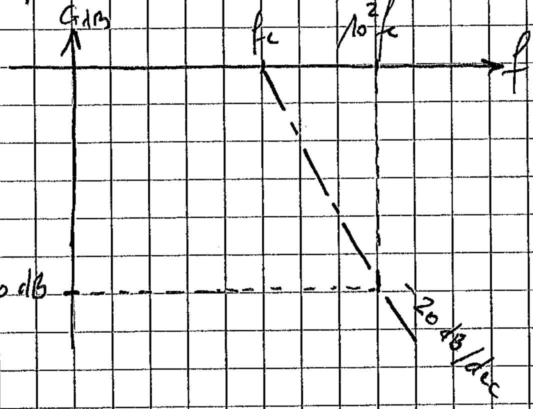
$\langle p_i \cdot v_i \rangle = \frac{1}{2}\frac{p_0^2}{Z_1}$

N.B. $\langle p_t v_t \rangle = \frac{1}{2}\mathrm{Re}(p_t v_t^*) = \frac{1}{2}\mathrm{Re}\left(t\, p_i \cdot \frac{t^* p_i^*}{Z_1}\right) = \frac{1}{2Z_1}|t\, p_i|^2$

$\rightarrow \boxed{T = |t|^2 = \frac{1}{1+\left(\frac{\omega\sigma}{2Z_1}\right)^2}}$

3\. $G_{dB} = 10\log(T) = -10\log\left(1+\left(\frac{\omega\sigma}{2Z_1}\right)^2\right)$ filtre passe bas du 1er ordre

$\omega_c = \frac{2Z_1}{\sigma} \rightarrow \boxed{f_c = \frac{Z_1}{\pi\sigma}}$



$G_{dB}(10^2 f_c) = -40$ dB.

avec $10^2 f_c = 200$ Hz $\rightarrow \underline{f_c = 2 \text{ Hz}}$

$\rightarrow \boxed{\sigma = \frac{Z_1}{\pi f_c} = \frac{\rho c}{\pi f_c}}$ où $\sigma = \rho a$

$\rightarrow \sigma = 65$ kg m^{-2} $\rightarrow a = 5,4$ cm.

$\lambda = \frac{c}{f} = 1,7$ m $\gg a$ donc hypothèse vérifiée.