P0218 Propagation d'une onde sonore dans un tuyau déformable

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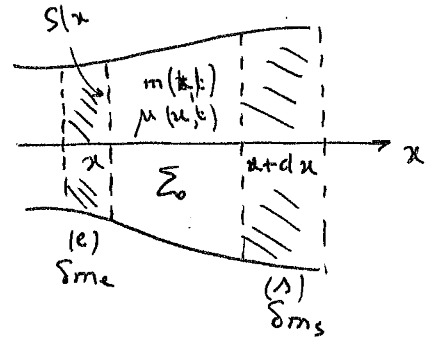

* $m_{\Sigma_0}(t+dt) - m_{\Sigma_0}(t) = dt \frac{\partial m}{\partial t} = \frac{\partial \mu S}{\partial t}\, dx\, dt$

* système fermé Σ_f.

à t : $m_{\Sigma_f}(t) = m_{\Sigma}(t) + \delta m_e = m_{\Sigma}(t) + \mu(x) S(x) v(x) dt$

à $t+dt$: $m_{\Sigma_f}(t+dt) = m_{\Sigma}(t+dt) + \delta m_s$

$= m_{\Sigma}(t+dt) + \mu(x+dx).S(x+dx).v(x+dx)dt$

$\rightarrow D m_{\Sigma_f} = 0 = D m_{\Sigma_0} + D(\mu S v)\, dt$

$\div\, dt\, dx$

$$\boxed{0 = \frac{\partial \mu S}{\partial t} + \frac{\partial}{\partial x}(\mu S v)}$$

$$\boxed{\mu_0 \frac{\partial S_1}{\partial t} + S_0 \frac{\partial \mu_1}{\partial t} + \mu_0 S_0 \frac{\partial v}{\partial x} = 0}$$ au 1er ordre.

* Or $\underline{\mu_1 = \mu_0 \chi_0 p_1}$ et $S = S_0 + p_1 \left(\frac{dS}{dp}\right)_1 = S_0 + D_0 p_1 S_0$ au 1er ordre.

$\Rightarrow \underline{S_1 = D_0 S_0 p_1}$

D'où $\mu_0 D_0 S_0 \frac{\partial p_1}{\partial t} + \mu_0 S_0 \chi \frac{\partial p_1}{\partial t} + \mu_0 S_0 \frac{\partial v}{\partial x} = 0$

$\mu_0 S_0 \left[(D_0 + \chi)\frac{\partial p_1}{\partial t} + \frac{\partial v}{\partial x}\right] = 0 \quad \Rightarrow \quad \boxed{(D_0 + \chi_0)\frac{\partial p_1}{\partial t} + \frac{\partial v}{\partial x} = 0}$ (a)

2. Euler linearisée : $\mu_0 \frac{\partial v}{\partial t} = - \frac{\partial p_1}{\partial x}$ (b)

$\frac{\partial (a)}{\partial t}$: $(D_0 + \chi_0)\frac{\partial^2 p_1}{\partial t^2} = - \frac{\partial^2 v}{\partial t \partial x}$

$\frac{\partial (b)}{\partial x}$: $\mu_0 \frac{\partial^2 v}{\partial x \partial t} = - \frac{\partial^2 p_1}{\partial x^2}$

$\rightarrow \mu_0 (D_0 + \chi_0)\frac{\partial^2 p_1}{\partial t^2} = + \frac{\partial^2 p_1}{\partial x^2}$

$\hookrightarrow \boxed{\frac{\partial^2 p_1}{\partial x^2} - \mu_0 (D_0 + \chi_0) \frac{\partial^2 p_1}{\partial t^2} = 0} \quad \rightarrow \quad \boxed{1/c^2 = \mu_0 (D_0 + \chi_0)}$

3 - $c = \dfrac{1}{\sqrt{\mu_0 (D_0 + \chi_0)}}$

$\left(c = \dfrac{c_{eau}}{\sqrt{1 + \frac{D_0}{\chi_0}}} \quad \text{car } c_{eau} = \dfrac{1}{\sqrt{\mu_0 \chi_0}} \right)$

$\frac{1}{c^2} = \mu_0 D_0 + \mu_0 \chi_0$

$= \mu_0 D_0 + \frac{1}{c_{eau}^2}$

$c = \dfrac{1}{\sqrt{\mu_0 D + \frac{1}{c_{eau}^2}}}$

$c_{métal} = 1,4 \cdot 10^3 \text{ m s}^{-1}$

$c_{sang} = 5,0 \text{ m s}^{-1}$

4 - * Pas d'onde venant de l'∞ donc seule existe l'onde progressive sens $x \nearrow$.

→ $v(x,t) = v(0, t - \frac{x}{c})$

Au premier ordre $Q_m = \mu_0 S_0 v(0,t)$

D'où $\begin{cases} v(x,t) = \frac{1}{\mu_0 S_0} Q_m(t - \frac{x}{c}) & \text{pour } x < ct \\ v(x,t) = 0 & \text{pour } x > ct \end{cases}$

* Euler : $\mu_0 \frac{\partial v}{\partial t} = -\frac{\partial p_1}{\partial x}$ → $\frac{1}{S_0}\frac{\partial Q_m}{\partial t} = -\frac{\partial p_1}{\partial x}$

or $\frac{\partial Q_m}{\partial t} = -c\frac{\partial Q_m}{\partial x}$ → $-\frac{1}{S}c\frac{\partial Q_m}{\partial x} = -\frac{\partial p_1}{\partial x}$.

$\int$ → $\begin{cases} p_1 = \frac{c}{S_0} Q_m(t - \frac{x}{c}) + cte(t) & \text{pour } x < ct, \\ p_1 = 0 & \text{pour } x > ct \end{cases}$

↳ nulle car on ne s'intéresse qu'aux OPPH.

5 - * $p_1 \simeq \frac{c}{S_0} Q_m = \frac{c\,\mu_0}{S_0} Q_v$; $Q_v = 4{,}7\ L\,min^{-1} = 7{,}8\ 10^{-5}\ m^3 s^{-1}$

$\mu_0 = 1{,}0\ 10^3\ kg\,m^{-3}$

↳ $p_1 \simeq 4{,}8\ 10^3\ Pa \simeq 5\%\ P_0$ ⇒ $\frac{p_1}{P_0} \ll 1$ tout juste

* Si $D \searrow$, $c \nearrow$ → $p_1 \nearrow$ à débit constant : la "tension" artérielle $\nearrow$.