PO 114 _ Onde sonore dans une barre

a) $\dfrac{\partial^2 \xi}{\partial t^2} - c^2 \dfrac{\partial^2 \xi}{\partial x^2} = 0$ où $\boxed{c = \sqrt{\dfrac{E}{\rho}}}$ $\quad c = 1{,}6\ 10^3\ ms^{-1}$

$c_{air} = 3{,}4\ 10^2\ ms^{-1}$

b) $\xi(x,t) = f(x) \sin(\omega t - \varphi)$

On injecte la solution dans l'équation de propagation :

$-\omega^2 f(x) - c^2 f''(x) = 0 \Rightarrow f'' + \dfrac{\omega^2}{c^2} f(x) = 0$ Posons $k = \dfrac{\omega}{c}$.

Solution $\quad f(x) = A \cos(kx) + B \sin(kx)$

C.L. : $\quad f(0) = 0 \Rightarrow A = 0$

$f(L) = 0 \Rightarrow B \sin(kL) = 0 \Rightarrow kL = n\pi \Rightarrow k_n = \dfrac{n\pi}{L}$

$\omega_n = \dfrac{n\pi c}{L}$

$F(0,t) = ES \dfrac{\partial \xi}{\partial x}(0,t) = F_0 \cos(\omega t)\, \vec{e}_x$

$\hookrightarrow ES B k_n \sin(\omega t - \varphi) = F_0 \cos(\omega t) \Rightarrow \begin{cases} \varphi = \pi/2 \\ B = \dfrac{F_0}{ES k_n} \end{cases}$

$\rightarrow \boxed{\xi_n(x,t) = \dfrac{F_0}{ES k_n} \sin(k_n x) \cos(\omega_n t)}$

Fondamentale : $n = 1$: $\omega_1 = \dfrac{\pi c}{L} \longrightarrow f_1 = \dfrac{\omega_1}{2\pi} = \dfrac{c}{2L} = 8{,}0\ 10^2\ Hz$

audible.