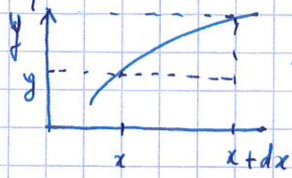
PO108 - Influence de la raideur d'une corde sur ses modes propres

1. $[Y] = \left[\frac{F}{S}\right] = ML^{-1}T^{-2}$; $[r] = L$; $[\gamma] = [Fd^2] = ML^3T^{-2}$

Eq aux dimensions $ML^3T^{-2} = M^p L^{q-p} T^{-2p}$ $\Rightarrow \begin{cases} p = 1 \\ q = 4 \end{cases} \Rightarrow \underline{\gamma = Y r^4}$

2. 

PFD $\begin{cases} 0 = T^{(x+dx)}\cos(\alpha(x+dx)) - T^{(x)}\cos\alpha(x) \\ \mu\, dx\, \ddot{y} = T\left[\underbrace{\sin(\alpha(x+dx))}_{\frac{\partial y}{\partial x}(x+dx)} - \underbrace{\sin(\alpha(x))}_{\frac{\partial y}{\partial x}(x)}\right] - \gamma\left(\frac{\partial^3 y}{\partial x^3}(x+dx) - \frac{\partial^3 y}{\partial x^3}(x)\right) \end{cases}$ $\Rightarrow \underline{T(x) = C^{te} = T}$

$$\Rightarrow \boxed{\frac{\partial^2 y}{\partial t^2} - \frac{T}{\mu}\frac{\partial^2 y}{\partial x^2} = -\frac{\gamma}{\mu}\frac{\partial^4 y}{\partial x^4}}$$

Les harmoniques de rang élevés sont plus modifiés car corde + cambrée.

3. On injecte la solution : $-\omega_n^2 + \frac{T}{\mu}\frac{n^2\pi^2}{L^2} = -\frac{\gamma}{\mu}\frac{n^4\pi^4}{L^4}$

Cas usuel : $\omega_{0n} = \frac{n\pi c}{L}$

$\xrightarrow{\text{ici}}$ $\boxed{\omega_n = \omega_{0n}\left(1 + \frac{\gamma}{T}\frac{n^2\pi^2}{L^2}\right)}$