P01006 - # CORDE DE PIANO

1 - $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{1}{c^2}\dfrac{\partial^2 z}{\partial t^2} = 0$ où $c = \sqrt{\dfrac{F}{\mu}}$

2 - Fixée aux extrémités $\Rightarrow$ solution stationnaire : $z(x,t) = z_0 \cos(\omega t + \phi)\cos(kx + \phi')$
où $k = \dfrac{\omega}{c}$

(CL) 1) $z(0,t) = 0 = z_0 \cos(\omega t + \phi)\cos\phi' \longrightarrow \phi' = \dfrac{\pi}{2}\ [\pi]$

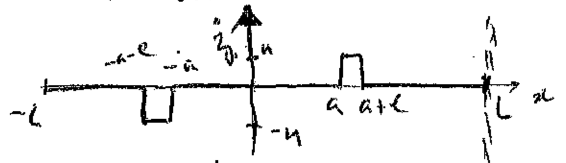
2) $z(L,t) = 0 = z_0 \cos(\omega t + \phi)\cos(kL + \phi') \rightarrow k_n = \dfrac{n\pi}{L} \longrightarrow \lambda_n = \dfrac{2L}{n}$

$\longrightarrow z = \sum_n A_n \cos\left(\dfrac{n\pi c}{L}t + \phi_n\right)\sin\left(\dfrac{n\pi}{L}x\right)$

3 - CI : 1) $z(x,0) = 0 = \sum_n A_n \cos\phi_n \sin\dfrac{n\pi}{L}x = 0 \rightarrow \phi_n = \dfrac{\pi}{2}\ [\pi]\ \forall n.$

2) $\left(\dfrac{\partial z}{\partial t}\right)_{t=0} = u \quad \forall x \in [a, a+e]$

$\left(\dfrac{\partial z}{\partial t}\right)_{t=0} = +\sum_n A_n \dfrac{n\pi c}{L} \cancel{\cos\left(\dfrac{n\pi c}{L}t\right)} \sin\left(\dfrac{n\pi x}{L}\right)$ = développement en série de Fourier de u étendu $\rightarrow f^n$ impaire



$u = \sum_n b_n \sin\left(n\dfrac{2\pi}{2L}x\right)$

(H.P.) où $b_n = \dfrac{2}{2L}\displaystyle\int_{-L}^{L} \dot{z}(x,0)\sin\left(n\dfrac{2\pi}{2L}x\right)dx = \dfrac{1}{L}\left[\int_{-a}^{-a-e} -u\sin(\ )dx + \int_a^{a+e} u\sin(\ )dx\right]$

$= \dfrac{2}{L}\displaystyle\int_a^{a+e} u\cdot\sin\left(\dfrac{n\pi x}{L}\right)dx = -\dfrac{2u}{n\pi}\left[\cos\left(\dfrac{n\pi}{L}(a+e)\right) - \cos\left(\dfrac{n\pi}{L}a\right)\right] = \dfrac{4u}{n\pi}\underbrace{\sin\left(\dfrac{n\pi e}{2L}\right)\sin\left(\dfrac{n\pi}{L}\left(a+\dfrac{e}{2}\right)\right)}_{\alpha}$

$\rightarrow \left.\dfrac{\partial z}{\partial t}\right)_{t=0} = \sum_n \dfrac{4\alpha u}{n\pi}\sin\left(\dfrac{n\pi x}{L}\right) = \sum_n A_n \dfrac{n\pi c}{L}\sin\dfrac{n\pi x}{L} \rightarrow A_n = \dfrac{4\alpha u L}{n^2\pi^2 c}$

4 a) $\langle \mathcal{E}_n \rangle = \left\langle \displaystyle\int \dfrac{1}{2}v_n^2\,dm \right\rangle = \left\langle \dfrac{\mu}{2}\int_0^L \left(\dfrac{\partial z_n}{\partial t}\right)^2 dx \right\rangle = \left\langle A_n^2 \left(\dfrac{n\pi c}{L}\right)^2 \dfrac{\mu}{2}\cos^2\left(\dfrac{n\pi c t}{L}\right)\int_0^L \sin^2\left(\dfrac{n\pi x}{L}\right)dx \right\rangle$

$\langle \mathcal{E}_n \rangle = \left(\dfrac{\mu\pi^2 c^2}{8L}\right) n^2 A_n^2$

b) $a = \dfrac{L}{7}$ et comme $e \ll a$

$\hookrightarrow A_n = \dfrac{4uL}{n^2\pi^2 c}\,\dfrac{n\pi e}{2L}\sin\left(\dfrac{n\pi}{7}\right) = \dfrac{2ue}{n\pi}\sin\left(\dfrac{n\pi}{7}\right)$

$\hookrightarrow \langle \mathcal{E}_n \rangle = \underbrace{\dfrac{\mu u^2 e^2}{2L}}_{\mathcal{E}_0}\sin^2\left(\dfrac{n\pi}{7}\right)$