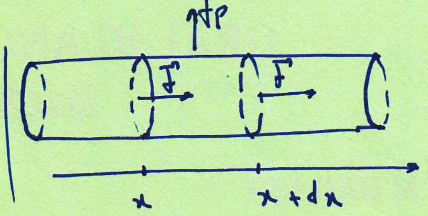
ORTH4 19 - <u>Ailette de refroidissement</u> (Navale - 2017 - Jérémy ROUDIN)

1 - Bilan sur tranche $[x, x+dx]$



1er principe : $dU = U(t+dt) - U(t) = \frac{\partial U}{\partial t} dt = \delta Q$

1re loi Joule : $dU = \rho c S\, dx\, dT$

$$\delta Q = \left(j_x(x,t) - j_x(x+dx,t)\right) S\, dt - h\left(T(x) - T_0\right) 2\pi a\, dx\, dt$$

$$= \left[-\frac{\partial j_x}{\partial x} \cdot \pi a^2 - 2\pi a h\left(T(x)-T_0\right)\right] dx\, dt \quad ; \quad j_x = -\lambda \frac{\partial T}{\partial x}$$

$$\Rightarrow \rho c\, \pi a^2 \frac{\partial T}{\partial t} = \left[+\lambda \pi a^2 \frac{\partial^2 T}{\partial x^2} - 2\pi a h\left(T - T_0\right)\right] dx\, dt$$

où $\underline{P_s = -h\left(T(x) - T_0\right)}$

2 - Régime stationnaire : $\frac{d^2 T}{dx^2} - \frac{2h}{\lambda a}(T - T_0) = 0$ soit $\alpha^2 = \frac{2h}{\lambda a}$

$$T(x) - T_0 = A e^{-\alpha x} + B e^{\alpha x}$$

Barreau long, i.e. $L \gg \frac{1}{\alpha} \Rightarrow \underline{B = 0}$

$$T(x) - T_0 = A e^{-\alpha x}$$

C.L. : $T(0) - T_0 = T_1 - T_0 = A \longrightarrow \boxed{T(x) = T_0 + (T_1 - T_0) e^{-\alpha x}}$

3 - $P = \iint P_s\, dS = -h \int_0^L \left(T(x) - T_0\right) \cdot 2\pi a\, dx = -2\pi a h (T_1 - T_0) \int_0^L e^{-\alpha x} dx$

$$P = +\frac{2\pi a h}{\alpha}(T_1 - T_0)\left(e^{-\alpha L} - 1\right) \quad \text{or} \quad L \gg \frac{1}{\alpha}$$

$$\Rightarrow \boxed{P \simeq -\frac{2\pi a h}{\alpha}(T_1 - T_0)}$$

4 - $P' = P_s(x=0) \times \pi a^2 = -h \pi a^2 (T_1 - T_0)$

Rendement : $\boxed{\frac{P}{P'} = \frac{a}{\alpha} = a\sqrt{\frac{\lambda a}{2h}}}$