ORTH 2k. Compresseur à 2 étages

1 - [redacted] ; 1ère loi de Joule : $\Delta h_{BP} = c_p (T_4 - T_1)$

loi de Laplace : $T_4 = T_1 \left(\frac{p_4}{p_1}\right)^{\frac{1-\gamma}{\gamma}} = T_1 \, a^{(\gamma-1)/\gamma}$

$\Rightarrow$ $\boxed{w = \Delta h = c_p T_1 \left(a^{\gamma-1/\gamma} - 1\right)}$ $\quad \underline{w = 4{,}5 \cdot 10^6 \text{ kJ kg}^{-1}}$

2 - $\boxed{w_{BP} = \Delta h_{BP} = c_p (T_2 - T_1) = c_p T_1 \left(\lambda^{\gamma-1/\gamma} - 1\right)}$

$w_{HP} = \Delta h_{HP} = c_p (T_4 - T_3)$ or $T_3 = T_1$; $p_3 = p_2$

$w_{HP} = c_p (T_4 - T_1)$; Laplace : $T_1^{\gamma} \, p_2^{1-\gamma} = T_4^{\gamma} \, p_4^{1-\gamma}$

$\Rightarrow T_4 = T_1 \left(\frac{p_2}{p_4}\right)^{1-\gamma/\gamma} = T_1 \left(\frac{\lambda}{a}\right)^{(1-\gamma)/\gamma}$

$\Rightarrow$ $\boxed{w_{HP} = c_p T_1 \left(\left(\frac{\lambda}{a}\right)^{\frac{1-\gamma}{\gamma}} - 1\right)}$

$\boxed{w_{tot} = w_{BP} + w_{HP} = c_p T_1 \left(\lambda^{(\gamma-1)/\gamma} + \left(\frac{\lambda}{a}\right)^{\frac{1-\gamma}{\gamma}} - 2\right)}$

3 - $\frac{dw_{tot}}{d\lambda} = c_p T_1 \, \frac{\gamma-1}{\gamma} \left(\lambda^{-1/\gamma} - \frac{\lambda^{(1-2\gamma)/\gamma}}{a^{(1-\gamma)/\gamma}}\right) = 0 \quad \dots \rightarrow \boxed{\lambda = \sqrt{a} = 5}$

4 - $T_2 = T_1 \, \lambda^{(\gamma-1)/\gamma} = 475 \text{ K}$

$T_4 = T_1 \left(\frac{\lambda}{a}\right)^{(1-\gamma)/\gamma} = T_1 \, \lambda^{(\gamma-1)/\gamma}$ car $\frac{\lambda}{a} = \frac{1}{\lambda}$ $\Rightarrow T_4 = 475 \text{ K}$

5 - Echangeur double flux calorifugé

$\dot{Q} = 0 = d_m c_p (T_3 - T_2) + \dot{d}_{eau} c_{eau} (T_5 - T_0)$; $T_5 = 293 \text{ K}$

$\Rightarrow$ $\boxed{d_{eau} = d_m \, \frac{c_p}{c_{eau}} \, \frac{T_2 - T_3}{T_5 - T_0}}$ $\quad \underline{d_{eau} = 5{,}4 \text{ kg s}^{-1}}$

NB : $w_{tot\,min} = 2 c_p T_1 \left(\lambda^{\frac{\gamma-1}{\gamma}} - 1\right)$

$\lambda = 5$, $a = 25 \Rightarrow \frac{\lambda}{a} = \frac{1}{\lambda}$

$\Rightarrow \frac{w_{tot\,min}}{w} = 2 \, \frac{\lambda^{\frac{\gamma-1}{\gamma}} - 1}{a^{\frac{\gamma-1}{\gamma}} - 1} = 0{,}77$