ORME 324 - Ecoulement dans un tuyau à section rectangulaire

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1 - * "Faible" Re ⇒ écoulement laminaire : $\vec{v} = v(z,y)\,\vec{e_x}$

* Ecoulement incompressible → $\operatorname{div}\vec{v} = 0$: $\dfrac{\partial v_x}{\partial x} = 0$.

* $b << w$ → invariance selon y : $\vec{v} = v(z)\,\vec{e_x}$

2. Navier-Stokes en régime stationnaire : $\underbrace{(\vec{v}.\vec{\text{grad}})\vec{v}}_{=0} = \rho\vec{g} - \vec{\text{grad}}\,p + \eta\Delta\vec{v}$

$$\begin{cases} 0 = -\dfrac{\partial p}{\partial x} + \eta\dfrac{\partial^2 v}{\partial z^2} \\ 0 = -\dfrac{\partial p}{\partial y} \\ 0 = +\rho g - \dfrac{\partial p}{\partial z} \end{cases} \Rightarrow \frac{\partial}{\partial x}\frac{\partial p}{\partial z} = 0 = \frac{\partial}{\partial z}\left(\frac{\partial p}{\partial x}\right) \rightarrow \frac{\partial p}{\partial x} \text{ indp}^t \text{ de } z.$$

$$\Rightarrow \frac{d^2 v}{dz^2} = -\frac{G}{\eta} \longrightarrow \underline{v(z) = -\frac{1}{2}\frac{G}{\eta}z^2 + Az + B}$$

C.L : Adhérence aux parois : $v\left(-\frac{b}{2}\right) = 0$ ⇒ $0 = -\frac{1}{8}\frac{G}{\eta}b^2 - \frac{Ab}{2} + B$

$v\left(\frac{b}{2}\right) = 0$; $0 = -\frac{1}{8}\frac{G}{\eta}b^2 + \frac{Ab}{2} + B$

↳ $\underline{A = 0}$ et $B = \frac{1}{8}\frac{G}{\eta}b^2$ ⇒ $\boxed{v(z) = \frac{1}{2}\frac{G}{\eta}\left(\frac{b^2}{4} - z^2\right)}$

3 - $$\bar{v}_x = \frac{D_v}{wb} = \frac{1}{wb}\iint_{w\,b} v\,dy\,dz = \frac{1}{\eta b}\left[\frac{1}{2}G\frac{b^2}{4}z - \frac{Gz^3}{6}\right]_{-b/2}^{b/2}$$

$$\bar{v}_x = \frac{Gb^2}{\eta 8} - \frac{Gb^2}{24\eta} \Rightarrow \boxed{\bar{v}_x = \frac{Gb^2}{12\eta}}$$

$$\frac{\Delta p}{D_v} \overset{\text{def}}{=} R_H \;;\; G = \frac{\Delta p}{L} \;;\; D_v = bw\bar{v}_x$$

$$R_H = \frac{\Delta p\;12\eta}{b.w.\frac{\Delta p}{L}b^2} \Rightarrow \boxed{R_H = \frac{12\eta}{wb^3}}$$