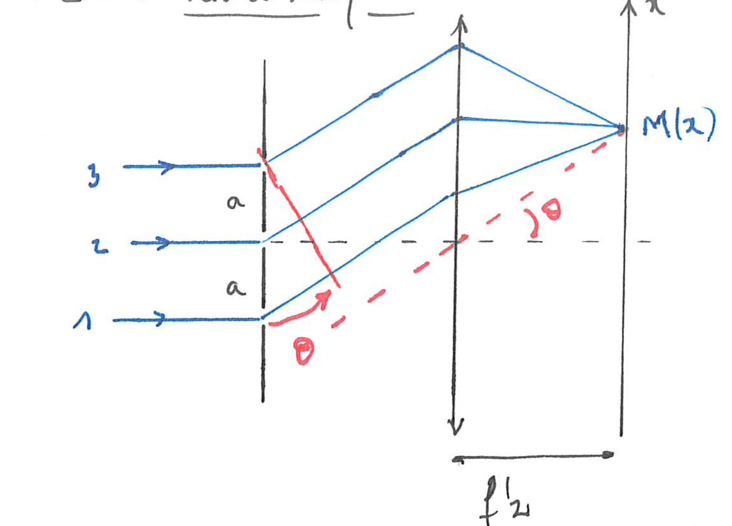
OP 210. Fentes triples



Différence de marche entre 2 ondes consécutives

$$\delta = \frac{a x}{f'_2}$$

$$\varphi = \frac{2\pi}{\lambda}\delta = 2\pi\frac{a x}{f'_2}$$

× Vibration résultant de la superposition des 3 ondes cohérentes en M :

$$\underline{s} = s_0 e^{i\omega t} + s_0 e^{i(\omega t - \varphi)} + s_0 e^{i(\omega t - 2\varphi)}$$

$$\underline{s} = s_0 e^{i\omega t}\left(1 + e^{-i\varphi} + e^{-2i\varphi}\right)$$

× Intensité lumineuse : $I = \underline{s}.\underline{s}^* = s_0^2\left(1 + e^{-i\varphi} + e^{-2i\varphi}\right)\left(1 + e^{i\varphi} + e^{2i\varphi}\right)$

$$I = s_0^2\left(1 + e^{i\varphi} + e^{2i\varphi} + e^{-i\varphi} + 1 + e^{i\varphi} + e^{-2i\varphi} + e^{-i\varphi} + 1\right)$$

$$= s_0^2\left(3 + 2\left(e^{i\varphi} + e^{-i\varphi}\right) + e^{2i\varphi} + e^{-2i\varphi}\right)$$

$$= s_0^2\left(3 + 4\cos\varphi + 2\cos 2\varphi\right)$$

N.B : $\cos^2\varphi = \frac{1 + \cos 2\varphi}{2}$

$\hookrightarrow \cos 2\varphi = 2\cos^2\varphi - 1$

$$I = s_0^2\left(1 + 4\cos\varphi + 4\cos^2\varphi\right)$$

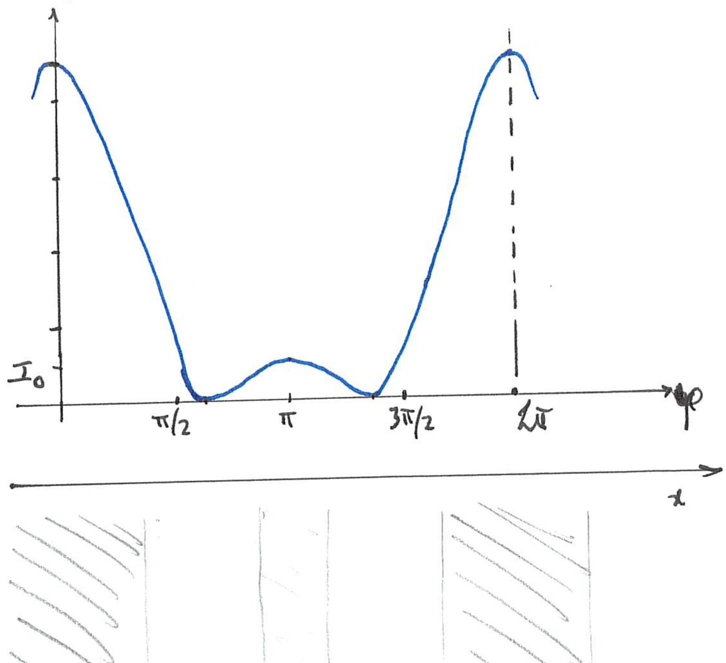
$\Rightarrow I(x)$ $\boxed{I = I_0\left(1 + 2\cos\varphi\right)^2}$ où $I_0 = s_0^2$

$I(0) = 9 I_0$

$I(\pi) = 1.\ I_0$

$I = 0$ pour $\varphi = 2{,}1$ rad

$\varphi = 4{,}2$ rad



Autre méthode

$$\underline{s} = s_0 e^{i\omega t}\sum_{k=0}^{2} e^{-ik\varphi} = s_0 e^{i\omega t}\frac{1 - e^{-3i\varphi}}{1 - e^{-i\varphi}}$$

$$\underline{s} = s_0 e^{i\omega t}\frac{e^{-3i\varphi/2}}{e^{-i\varphi/2}}\frac{\sin(3\varphi/2)}{\sin(\varphi/2)}$$

$$\boxed{\mathcal{E} = \underline{s}\,\underline{s}^* = s_0^2\frac{\sin^2(3\varphi/2)}{\sin^2(\varphi/2)}}$$

Cohérence des expressions

$$\sin\left(\frac{3\varphi}{2}\right) = \sin\left(\frac{\varphi}{2} + \varphi\right) = \sin\left(\frac{\varphi}{2}\right)\cos\varphi + \sin\varphi\cos\frac{\varphi}{2}$$

$$= \sin\frac{\varphi}{2}\left[\cos\varphi + 2\cos^2\frac{\varphi}{2}\right] = \sin\left(\frac{\varphi}{2}\right)\left[\cos\varphi + 1 + \cos\varphi\right]$$

$$\Rightarrow \boxed{\mathcal{E} = I_0\left(1 + 2\cos\varphi\right)^2}$$