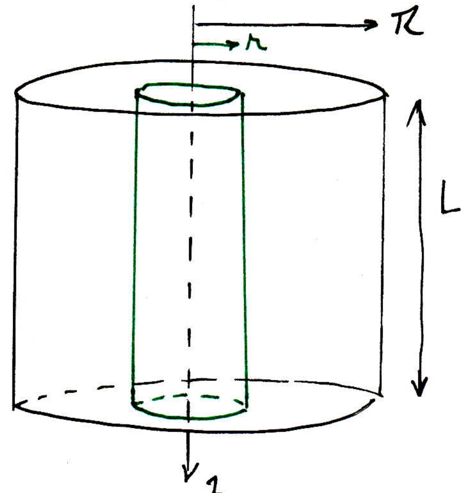
MF504 - Viscosimètre



1. Système : Colonne cylindrique de fluide, de hauteur L, de rayon r.

Bilan des forces extérieures :
- Poids : $\vec{P}$
- Force de cisaillement $\vec{F_{cis}}$
- Force pressantes $\vec{F_p}$

2. $\vec{P} = \mu \pi r^2 L \vec{g}$

$\vec{F_{cis}} = \eta \frac{\partial v}{\partial r} 2\pi r L \vec{e_z}$

$\vec{F_p} = (P_1 - P_2)\pi r^2 \vec{e_z}$

Loi de la quantité de mouvement en écoulement stationnaire

$\vec{0} = \vec{P} + \vec{F_{cis}} + \vec{F_p}$

Projection selon $\vec{e_z}$: $0 = \mu \pi r^2 L g + \eta \frac{dv}{dr} 2\pi r L + (P_1 - P_2)\pi r^2$

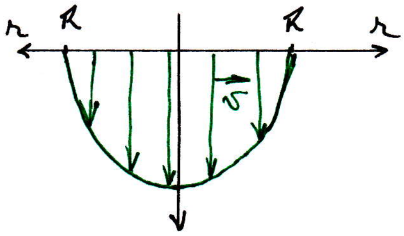
$\Rightarrow \boxed{\frac{dv}{dr} = k r}$ où $\boxed{k = -\frac{\mu g}{2\eta} - \frac{1}{2\eta L}(P_1 - P_2)}$ Rq : $k < 0$.

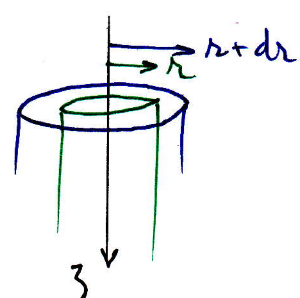
3. Solution : $v(r) = \frac{1}{2} k r^2 + A$

Condition limite : Adhérence aux parois du capillaire : $v(R) = 0$

$\Rightarrow 0 = \frac{1}{2} k R^2 + A \Rightarrow A = -\frac{1}{2} k R^2 \Rightarrow \boxed{v(r) = \frac{1}{2} k (r^2 - R^2)}$

4





5. $\boxed{dQ_v = v(r) \times 2\pi r dr}$

$Q_v = \int_0^R v(r) 2\pi r dr = \frac{1}{2} k \int_0^R (r^2 - R^2) 2\pi r dr$

$Q_v = \pi k \left(\frac{R^4}{4} - \frac{R^4}{2}\right) \rightarrow \boxed{Q_v = -\frac{\pi k R^4}{4}}$

$\Rightarrow \boxed{Q_v = \frac{\pi R^4 \mu g}{8\eta} + \frac{\pi R^4}{8\eta L}(P_1 - P_2)}$

loi de Poiseuille $R_H = \frac{P_1 - P_2}{Q_v} = \frac{8\eta L}{\pi R^4}$