* MF316 - Écoulement autour d'un cylindre en rotation

1 - On rappelle le champ de vitesse du tourbillon de Rankine $\vec{v} = \begin{cases} r\Omega\vec{e_\theta} \ ; \ r<R \\ \frac{R^2}{r}\Omega\vec{e_\theta} \ ; \ r>R \end{cases}$

A l'extérieur du tourbillon $\vec{v} = \frac{R^2}{r}\Omega\vec{e_\theta} = \overrightarrow{grad}\,\Phi_{tourbillon}$

$\frac{R^2}{r}\Omega\vec{e_\theta} = \frac{\partial\Phi_{to.}}{r\,\partial\theta} \longrightarrow \boxed{\Phi_{tour.} = R^2\Omega\theta}$

2 a) $U\vec{e_x} = \overrightarrow{grad}\,\Phi_U = \frac{\partial\Phi_U}{\partial x}\vec{e_x} \Rightarrow \boxed{\Phi_U = Ux = Ur\cos\theta}$

b) Condition limite : $v_r(r=R,\theta)=0$; $v_r = \frac{\partial\Phi}{\partial r} = U\cos\theta - \frac{A\cos\theta}{2\pi r^2}$

$v_r(R,\theta) = \cos\theta\left(U - \frac{A}{2\pi R^2}\right) = 0 \rightarrow \boxed{A = 2\pi R^2 U}$

$$\rightarrow \vec{v} = \begin{vmatrix} U\cos\theta\left(1-\frac{R^2}{r^2}\right) \\ -U\sin\theta\left(1+\frac{R^2}{r^2}\right) + \frac{R^2\Omega}{r} \\ 0 \end{vmatrix}$$

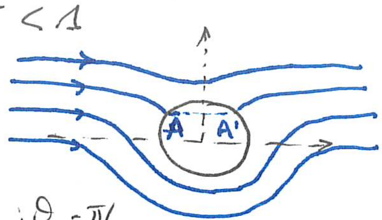
c) Points d'arrêt : $\vec{v} = \vec{0}$ $\quad v_r = 0 \Rightarrow \begin{cases} r = R \\ \theta = \pi/2 \ ; \ 3\pi/2 \end{cases}$

Posons $\Omega > 0$

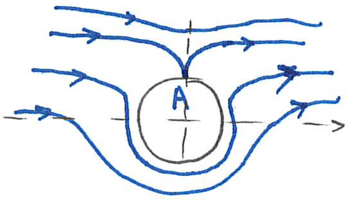
$$v_\theta = 0 \Rightarrow \begin{cases} r=R : \ -2U\sin\theta + R\Omega = 0 \\ \theta = \frac{\pi}{2} : \ -U\left(1+\frac{R^2}{r^2}\right) + \frac{R^2\Omega}{r} = 0 \\ \theta = \frac{3}{2} : \ U\left(1+\frac{R^2}{r^2}\right) + \frac{R^2\Omega}{r} = 0 \end{cases}$$

Pas de solution pour $\Omega>0$

$r=R$ $\quad \sin\theta = \frac{R\Omega}{2U}$: Pour $\Omega>0$: 2 solutions si $\frac{R\Omega}{2U} < 1$



1 solution si $\frac{R\Omega}{2U} = 1$: $\theta = \pi/2$



$r \neq R$, $\theta = \frac{\pi}{2}$ $\rightarrow \frac{R\Omega}{2U} > 1$

$-U\left(1+\frac{R^2}{r^2}\right) + \frac{R^2\Omega}{r} = 0$

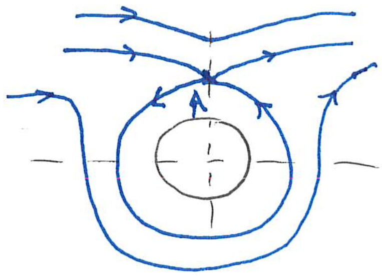
$Ur^2 + UR^2 - R^2 r\Omega = 0$

$\hookrightarrow r_\pm = \frac{1}{2U}\left(R^2\Omega \pm \sqrt{R^4\Omega^2 - 4U^2R^2}\right)$

$= \frac{R\Omega}{2U}\left(R \pm R\sqrt{1-\frac{4U^2}{R^2\Omega^2}}\right)$

$r_- < R$: non recevable

$r_+ > R$.



Posons $x = \frac{R\Omega}{2U} > 1$

$r_- = R\left(\frac{R\Omega}{2U} - \sqrt{\frac{R^2\Omega^2}{4U^2} - 1}\right) = R\left(x - \sqrt{x^2-1}\right)$

A-t-on $r_- < R$ soit $x - \sqrt{x^2-1} < 1 \longrightarrow x - 1 < \sqrt{x^2-1}$

$x^2+1-2x < x^2-1$

$x > 1$: oui

Donc $r_- < R$