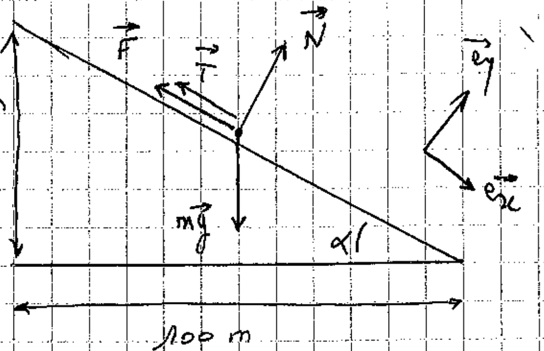
ME 310 - *Kilomètre lancé*



a- $\tan\alpha \overset{def}{=} 62{,}5\,\%$ → $\alpha = 0{,}586$ rad

$\alpha = 32{,}0°$

b- PFD ds réf terrestre galiléen

$$m\vec{\gamma} = m\vec{g} + \vec{N} + \vec{T} + \vec{F}$$

Projection : $\begin{cases} m\ddot{x} = mg\sin\alpha - T - \frac{1}{2}\rho C_x S\, v\, v_x \\ 0 = -mg\cos\alpha + N \end{cases}$ ⇒ $N = mg\cos\alpha$

Loi de Coulomb du glissement : $\mu = \frac{\|\vec{T}\|}{\|\vec{N}\|}$ ⇒ $T = mg\mu\cos\alpha$

⇒ $\ddot{x} = g(\sin\alpha - \mu\cos\alpha) - \frac{1}{2}\frac{\rho C_x S}{m} v\, v_x$ où $v_x = v = \|\vec{v}\|$.

Une fois la vitesse limite atteinte : $\ddot{x} = 0$

⇒ $a - b v_{lim}^2 = 0$ → $\boxed{v_{lim} = \sqrt{\frac{a}{b}}}$ $v_{lim} = 32\ ms^{-1}$

c- $\dot{v} = a - b v^2 = a\left[1 - \left(\frac{v}{v_{lim}}\right)^2\right]$

Posons $u = \frac{v}{v_{lim}}$ → $\boxed{\frac{du}{1-u^2} = \frac{a}{v_{lim}}dt}$

$\int\downarrow$ $\frac{1}{2}\ln\left|\frac{1+u}{1-u}\right| = \frac{a}{v_{lim}}t$ or $1-u > 0$ car $u < 1$

$\frac{1+u}{1-u} = e^{\frac{2a}{v_{lim}}t}$ → $u(t) = \frac{-1+e^{\frac{2a}{v_l}t}}{1+e^{\frac{2a}{v_l}t}}$

$\boxed{u = \frac{sh\left(\frac{a}{v_l}t\right)}{ch\left(\frac{a}{v_l}t\right)}}$ → $v(t) = v_{lim}\, th\left(\frac{a}{v_{lim}}t\right)$

$u = \frac{\dot{x}}{v_l} = \frac{sh(at/v_l)}{ch(at/v_l)}$ On intègre $\frac{x(t) - x(0)}{v_l} = \frac{v_l}{a}\ln\left(ch\left(\frac{at}{v_l}\right)\right)$

À $t = t_d$, $x(t_d) = d = 650$ m ⇒ $\frac{d}{v_l} = \frac{v_l}{a}\ln\left(ch\left(\frac{at_d}{v_l}\right)\right)$ → $\boxed{ch\left(\frac{at_d}{v_l}\right) = e^{ad/v_l^2}}$

$u = \frac{\sqrt{ch^2(at/v_l) - 1}}{ch(at/v_l)}$ ⇒ $\boxed{v(t_d) = v_l\frac{\sqrt{e^{2ad/v_l^2} - 1}}{e^{ad/v_l}}}$

A.N : $a = 5{,}26\ ms^{-2}$ → $v_{t_d} = 32\ ms^{-1} \simeq v_l$.

$\rho = 1{,}29\ kg\,m^{-3}$ à 273 K.

$F = 0{,}55$ kN ; N.B. $P = mg = 1{,}0$ kN