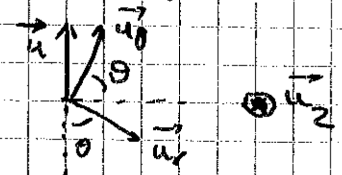
* ME228 - Anneau coulissant sur un cercle en rotation

1\. (C) : référentiel non galiléen en rotation uniforme / ref. du labo galiléen



B d F : $\vec{P} = -mg\vec{x} = -mg \begin{vmatrix} -\cos\theta \\ +\sin\theta \\ 0 \end{vmatrix}$

$\vec{N} = -N\vec{u_r}$

$\vec{N'} = -N\vec{u_z}$

$\vec{F_{ie}} = +m\Omega^2\vec{HM} = +m\Omega^2 a \begin{vmatrix} \sin^2\theta \\ \sin\theta\cos\theta \\ 0 \end{vmatrix}$

$\vec{F_{ic}} = -2m\vec{\Omega}\wedge\vec{v_r} = -2m \begin{vmatrix} -\Omega\cos\theta \\ +\Omega\sin\theta \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ a\dot\theta \\ 0 \end{vmatrix}$

$= +2m\Omega a\dot\theta\cos\theta\,\vec{u_z}$

P.F.D dans (C)

$m\vec{a_r} = \vec{N} + \vec{N'} + \vec{P} + \vec{F_{ie}} + \vec{F_{ic}}$

$$\begin{cases} -ma\dot\theta^2 = -N + mg\cos\theta + m\Omega^2 a\sin^2\theta \\ +ma\ddot\theta = -mg\sin\theta + m\Omega^2 a\sin\theta\cos\theta \\ 0 = -N' + 2m\Omega a\dot\theta\cos\theta \end{cases}$$

2\. Eq$^\theta$ du mouvement : $\ddot\theta = \Omega^2\sin\theta\cos\theta - \omega_0^2\sin\theta$

$$\boxed{\ddot\theta = \sin\theta\left(\Omega^2\cos\theta - \omega_0^2\right)}$$

Positions d'équilibre relatif : $\ddot\theta = 0$ si $\sin\theta = 0 \Rightarrow \theta = 0\,;\pi$.

ou $\cos\theta = \dfrac{\omega_0^2}{\Omega^2} = \dfrac{1}{\lambda}$ si $\lambda > 1$ ie $\Omega > \omega_0$

3\. $\ddot\theta = \omega_0^2 \underbrace{\sin\theta(\lambda\cos\theta - 1)}_{f(\theta)} \xrightarrow{DL} f(\theta) \simeq \cancel{f(\theta_0)} + \varepsilon\dfrac{df}{d\theta}(\theta_0)$

par def$^\circ$ de pos. d'eq.

$\ddot\theta = \ddot\varepsilon$

$\ddot\varepsilon - \varepsilon \times \omega_0^2 f'_0 = 0$ si $f'_0 < 0$: eq$^\circ$ oscillateur harmonique → EQ STABLE

$\underline{\omega = \omega_0\sqrt{|f'_0|}}$ si $f'_0 > 0$: EQ INSTABLE

$f'_0 = \lambda(\cos^2\theta_0 - \sin^2\theta_0) - \cos\theta_0 = 2\lambda\cos^2\theta_0 - \lambda - \cos\theta_0$

- $\theta_0 = 0 \to f'_0 = \lambda - 1$
- $\theta_0 = \pi \to f'_0 = \lambda + 1$
- $\cos\theta = \dfrac{1}{\lambda} \to f'_0 = \dfrac{1}{\lambda} - \lambda$

4 - $E_c = \frac{1}{2} m v_a^2 = \frac{1}{2} m \|\vec{v_r} + \vec{v_e}\|^2 = \frac{1}{2} m \|a\dot{\theta}\,\vec{u_r} + \vec{\Omega} \wedge a\vec{u_r}\|^2$

$= \frac{1}{2} m \left(a\dot{\theta}\,\vec{u_r} + \Omega a \sin\theta\,\vec{u_z}\right)^2 \rightarrow \underline{E_c = \frac{1}{2} m a^2 \left(\dot{\theta}^2 + \Omega^2 \sin^2\theta\right)}$

$\underline{E_c' = \frac{1}{2} m v_r^2 = \frac{1}{2} m a^2 \dot{\theta}^2}$

$E_{pp} = mgZ + Cte = mg(a - a\cos\theta) + Cte = -mga\cos\theta + Cte$

$E_p(F_{ie}) = -\frac{1}{2} m \Omega^2 r^2 = -\frac{1}{2} m \Omega^2 a^2 \sin^2\theta$

$E_m = E_c + E_{pp} = \frac{1}{2} m a^2 \left(\dot{\theta}^2 + \Omega^2 \sin^2\theta\right) - mga\cos\theta$

$\frac{dE_m}{dt} = m a^2 \dot{\theta}\ddot{\theta} + \dot{\theta}\, m a^2 \Omega^2 \cos\theta \sin\theta + mga\dot{\theta}\sin\theta$

or $\ddot{\theta} = \sin\theta\left(\Omega^2\cos\theta - \omega_0^2\right)$

$\frac{dE_m}{dt} = m a^2 \dot{\theta}\,\Omega^2 \sin\theta\cos\theta - \cancel{m a^2 \dot{\theta} \sin\theta\, \omega_0^2} + m a^2 \dot{\theta}\,\Omega^2 \cos\theta\sin\theta$

$+ \cancel{mga\dot{\theta}\sin\theta}$

$= 2 m a^2 \dot{\theta}\,\Omega^2 \cos\theta \sin\theta = -\vec{F_{ic}} \cdot \vec{v_a} = \vec{N'} \cdot \vec{v_a}$

Système non conservatif ds R

$E_m' = E_c' + E_{pp} + E_p(F_{ie})$

$= \frac{1}{2} m a^2 \dot{\theta}^2 - mga\cos\theta - \frac{1}{2} m \Omega^2 a^2 \sin^2\theta$

$\frac{dE_m'}{dt} = m a^2 \dot{\theta}\ddot{\theta} + mga\dot{\theta}\sin\theta - m\Omega^2 a^2 \dot{\theta} \cos\theta \sin\theta$

$= \cancel{m a^2 \dot{\theta}\,\Omega^2 \sin\theta\cos\theta} - \cancel{m a^2 \omega_0^2 \dot{\theta}\sin\theta} + \cancel{mga\dot{\theta}\sin\theta} - \cancel{m\Omega^2 a^2 \dot{\theta}\sin\theta\cos\theta}$

$= 0$

Système conservatif ds R'