ME106. Toboggan hélicoïdal.

a) $\vec{OM} = r\vec{u_r} + z\vec{u_z} = \underbrace{r}_{R}\cos\theta\,\vec{u_x} + \underbrace{r}_{R}\sin\theta\,\vec{u_y} + \underbrace{z}_{-b\theta}\vec{u_z}$

$\Rightarrow \boxed{\vec{OM} = R\vec{u_r} - b\theta\,\vec{u_z}}$

b) $\vec{v}_{\mathcal{R}}(M) = R\dot\theta\,\vec{u_\theta} + b\dot\theta\,\vec{u_z}$; $\boxed{v = \dot\theta\sqrt{R^2+b^2}}$

↳ réf lié au sol.

c) $-c = -b(\theta+2\pi) + b\theta = -2\pi b \rightarrow \underline{c = 2\pi b}$ $\quad \underline{b = \frac{c}{2\pi} = 0{,}32\,m}$

d) Mvt uniformément varié $\Rightarrow v = at = \dot\theta\sqrt{R^2+b^2} \Rightarrow \boxed{\dot\theta = \frac{a}{\sqrt{\ }}\,t}$

* $\int_0^{3\times 2\pi} d\theta = \frac{a}{\sqrt{\ }}\int_0^{\tau} t\,dt \longrightarrow 6\pi = \frac{a}{\sqrt{\ }}\frac{\tau^2}{2}$

* $v_3 = a\tau \Rightarrow a = \frac{v_3}{\tau} \longrightarrow 6\pi = \frac{v_3}{2\sqrt{\ }}\tau$

$\Rightarrow \boxed{\tau = \frac{12\pi\sqrt{R^2+b^2}}{v_3}}$ $\quad \underline{\tau = 16\ s}$

e) $\vec{a} = -R\dot\theta^2\vec{u_r} + R\ddot\theta\,\vec{u_\theta} + \ddot{z}\,\vec{u_z}$

$= -v^2\frac{R}{R^2+b^2}\vec{u_r} + \frac{Ra}{\sqrt{\ }}\vec{u_\theta} + \frac{ba}{\sqrt{\ }}\vec{u_z}$

$\|\vec{a}\| = \sqrt{v^4\frac{R^2}{(R^2+b^2)^2} + \frac{R^2a^2}{R^2+b^2} + \frac{b^2a^2}{R^2+b^2}}$

au 3e tour

$a_3 = \sqrt{v_3^4\frac{R^2}{(R^2+b^2)^2} + \frac{R^2v_3^2}{\tau^2(R^2+b^2)} + \frac{b^2v_3^2}{\tau^2(R^2+b^2)}}$

$\boxed{a_3 = \frac{1}{R^2+b^2}\sqrt{v_3^4R^2 + \frac{v_3^2}{\tau^2}(R^2+b^2)^2}}$ $\quad a_3 = 29\ ms^{-2}$

$\underline{a_3 = 2{,}9\ g}$