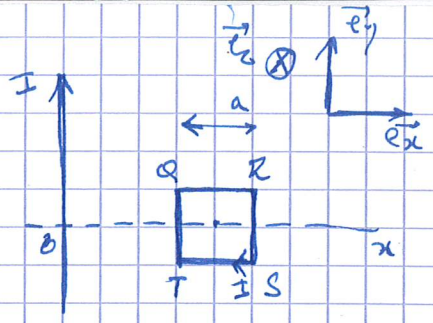
EM 10 - Action d'un fil sur un cadre



1) $\vec{B}_{fil} = \frac{\mu_0 I}{2\pi r} \vec{e_\theta}$ $\quad \vec{e_\theta} = \vec{e_z}$ dans le plan de figure.

$$\vec{F} = I \left(\int_{x-\frac{a}{2}}^{x+\frac{a}{2}} -\frac{\mu_0 I}{2\pi x} \vec{e_z} \wedge dx \vec{e_x} + \int_{y=-\frac{a}{2}}^{y=\frac{a}{2}} -\frac{\mu_0 I}{2\pi (x+\frac{a}{2})} \vec{e_z} \wedge dy \vec{e_y} + \int_{x+\frac{a}{2}}^{x-\frac{a}{2}} -\frac{\mu_0 I}{2\pi x} \vec{e_z} \wedge dx \vec{e_x} + \int_{y=\frac{a}{2}}^{y=-\frac{a}{2}} -\frac{\mu_0 I}{2\pi (x-\frac{a}{2})} \vec{e_z} \wedge dy \vec{e_y} \right)$$

$$\vec{F} = -\frac{\mu_0 I^2 a}{2\pi} \left(\frac{1}{x-\frac{a}{2}} - \frac{1}{x+\frac{a}{2}} \right) \vec{e_x} = -\frac{\mu_0 I^2 a}{2\pi} \left(\frac{a}{x^2 - \frac{a^2}{4}} \right) \vec{e_x}$$

Or $x \gg a \longrightarrow \quad \vec{F} \simeq -\frac{\mu_0 I^2 a^2}{2\pi x^2} \vec{e_x}$

2) $\vec{\mathcal{M}} = IS \vec{e_z} = I a^2 \vec{e_z}$

$$E_p \simeq -\vec{\mathcal{M}} \vec{B}(x) = -\frac{\mu_0 I^2 a^2}{2\pi x} e_z \quad ; \quad \vec{F} = -\frac{dE_p}{dx} \vec{e_x} = -\frac{\mu_0 I^2 a^2}{2\pi x^2} \vec{e_x}$$