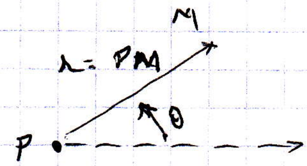
EM 310 - Ligne bifilaire



1- Symétries $(M, \vec{e_r}, \vec{e_\theta})$; $(M, \vec{e_r}, \vec{e_z})$

$$\hookrightarrow \vec{E}_p(M) = E_p(M) \cdot \frac{\vec{PM}}{PM}$$

Invariances : Rotation d'angle θ qcque ; Translation selon z qcque $\Big) \to \vec{E}(r, \theta, z)$

Surface de Gauss : cylindre (Pz, r), de hauteur h.

Th. de Gauss : $\oint\!\!\oint \vec{E}\, d\vec{S} = E(r)\, 2\pi r h$; $\frac{Q_{int}}{\varepsilon_0} = \frac{\lambda h}{\varepsilon_0}$ $\Big\} \to \boxed{\vec{E} = \frac{\lambda}{2\pi r \varepsilon_0} \vec{e_r}}$

$$\to V = \frac{-\lambda}{2\pi\varepsilon_0} \ln r + Cte \longrightarrow V_p(M) = \frac{-\lambda}{2\pi\varepsilon_0} \ln(PM) + Cte.$$

2- * Superposition : $V(M) = V_p(M) + V_N(M) = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{PM}{NM}\right) + K$.

Or $V(O) = 0 \Rightarrow K = 0$ $\hookrightarrow \boxed{V(M) = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{PM}{NM}\right)}$

* $PM = \sqrt{\vec{PM}^2} = \sqrt{(\vec{PO} + \vec{OM})^2} = \left(\left(\frac{D}{2}\right)^2 + r^2 + 2\,\vec{PO}\cdot\vec{OM}\right)^{1/2}$

$$= r\left(1 + \frac{D^2}{4r^2} - \frac{D}{r}\cos\theta\right)^{1/2} \underset{DL1}{\simeq} r\left(1 - \frac{D\cos\theta}{2r}\right)$$

$$NM = \ldots / \ldots \simeq r\left(1 + \frac{D\cos\theta}{2r}\right)$$

$$\Rightarrow V(M) = \frac{-\lambda}{2\pi\varepsilon_0} \ln\left(\frac{1 - D\cos\theta/2r}{1 + D\cos\theta/2r}\right) \underset{DL}{\simeq} \frac{\lambda}{2\pi\varepsilon_0 r} D\cos\theta$$

$$\Rightarrow \boxed{V(M) \simeq \frac{\lambda D}{2\pi\varepsilon_0 r} \cos\theta}$$

3- $\vec{E} = -\vec{grad}\, V = -\frac{\lambda D}{2\pi\varepsilon_0 r} \left| \begin{array}{l} -\frac{\cos\theta}{r} \\ -\frac{\sin\theta}{r} \end{array} \right.$

4- Au pied de la ligne en $O'(r, -\frac{\pi}{2})$: $\boxed{\vec{E}_0 = \frac{-\lambda D}{2\pi\varepsilon_0 h^2} \vec{u_\theta}}$

5- Plans de symétries : $(O, \vec{e_x}, \vec{e_z})$; $(O, \vec{e_x}, \vec{e_y})$

anti-sym : $(O, \vec{e_y}, \vec{e_z})$

7- $E_0 \simeq \frac{\Delta V}{\Delta x} \simeq \frac{400}{2} = 200\ V/m < 5\ kV/m$.