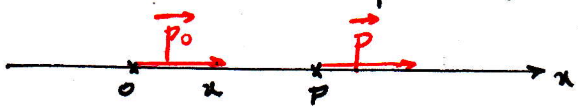
EM304 - Interaction dipôle-dipôle

1a)



$\vec{E}_0(P) = \frac{p_0}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\,\vec{e_r} + \sin\theta\,\vec{e_\theta}\right)$ en coordonnées sphériques

Ici, $r = x$, $\theta = 0 \Rightarrow \vec{e_r} = \vec{e_x}$

$\Rightarrow \boxed{\vec{E}_0(P) = \frac{2p_0}{4\pi\varepsilon_0 x^3}\vec{e_x}}$

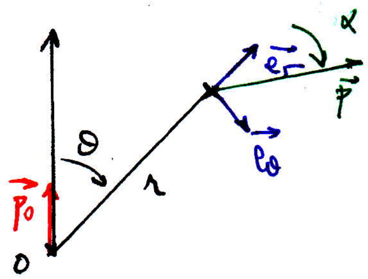
$\vec{F}(P) = (\vec{p}\cdot\vec{\text{grad}})\vec{E}_0(P) = p\frac{d}{dx}\left(\frac{2p_0}{4\pi\varepsilon_0 x^3}\right)\vec{e_x} \Rightarrow \boxed{\vec{F}(P) = -\frac{3pp_0}{2\pi\varepsilon_0 x^4}\vec{e_x}}$

Autre méthode : $\vec{F}(P) = -\vec{\text{grad}}\left(-\vec{p}\cdot\vec{E}_0(P)\right)$

$= \vec{\text{grad}}\left(\frac{2pp_0}{4\pi\varepsilon_0 x^3}\right) \rightarrow \vec{F}(P) = -\frac{3pp_0}{2\pi\varepsilon_0 x^4}\vec{e_x}$

b) $\boxed{\Delta E = E_p(\theta=\pi) - E_p(\theta=0) = +\frac{pp_0}{\pi\varepsilon_0 x^3}}$

2a)



$E_p = -\vec{p}\cdot\vec{E}_0(P)$ où $\vec{p} = p\left(\cos\alpha\,\vec{e_r} + \sin\alpha\,\vec{e_\theta}\right)$

$\Rightarrow \boxed{E_p = -\frac{p_0 p}{4\pi\varepsilon_0 r^3}\left(2\cos\theta\cos\alpha + \sin\theta\sin\alpha\right)}$

b) $\frac{dE_p}{d\alpha} = -\frac{p_0 p}{4\pi\varepsilon_0 r^3}\left(-2\cos\theta\sin\alpha + \sin\theta\cos\alpha\right) = 0 \Rightarrow \boxed{\tan\alpha = \frac{1}{2}\tan\theta}$