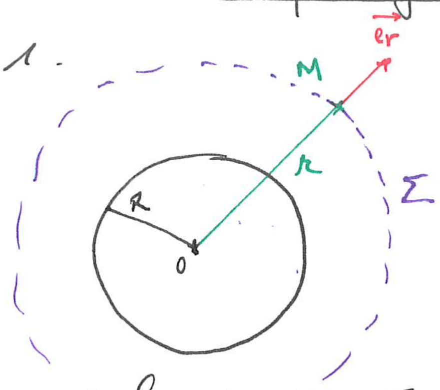
EM232 - Champ de gravitation terrestre

1.



a) Symétries : Tout plan contenant (OM) est plan de symétrie $\Rightarrow \vec{\Gamma_T}(M) = \Gamma_T \vec{e_r}$

Invariances, par rotation quelconque de centre O $\Rightarrow \vec{\Gamma_T}(M) = \vec{\Gamma_T}(r)$

d'où $\boxed{\vec{\Gamma_T}(M) = -\Gamma_T(r)\vec{e_r}}$ NB : $\Gamma_T = \|\vec{\Gamma_T}\|$.

Surface de Gauss Σ : Sphère (O, r)

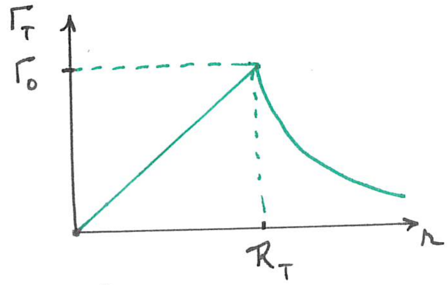
Théorème de Gauss de la gravitation : $\oiint_\Sigma \vec{\Gamma_T}\, d\vec{S} = -4\pi G m_{int}$.

$\oiint \vec{\Gamma_T}\, dS = -\Gamma_T(r) \cdot 4\pi r^2$

* $r < R$: $m_{int} = \dfrac{3M}{4\pi R^3} \times \dfrac{4}{3}\pi r^3 = M\dfrac{r^3}{R^3}$ $\Rightarrow \boxed{\Gamma_T(r) = \dfrac{MGr}{R^3}}$

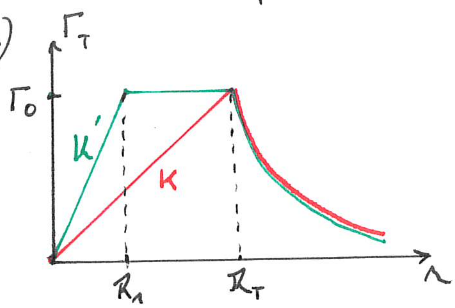
* $r > R$: $m_{int} = M$ $\Rightarrow \boxed{\Gamma_T(r) = \dfrac{GM}{r^2}}$ Champ de Newton

b)



c) $\boxed{\Gamma_0 = \Gamma_T(R_T) = \dfrac{GM}{R_T^2}}$ $\Gamma_0 = 9{,}80\ m\,s^{-2}$

2 - a)



b) $\Gamma_T(r) = \dfrac{\frac{4\pi}{3}\, M}{\frac{4}{3}\pi R^3}\, G r = \rho\, \dfrac{4}{3}\pi G r$

La pente de $\Gamma_T(r)$ est proportionnelle à ρ.

$\Gamma_T(r) = K r$

$\rightarrow K = \dfrac{\Gamma_0}{R_T}$; $K' = \dfrac{\Gamma_0}{R_1}$ $\Rightarrow \dfrac{K'}{K} = \dfrac{R_T}{R_1}$

$\Rightarrow \rho' = \rho\dfrac{R_T}{R_1}$ $\rho = 5{,}50\ 10^3\ kg\ m^{-3}$ $\Rightarrow \rho' = 10{,}0\ 10^3\ kg.m^{-3}$

c) Théorème de Gauss local de la gravitation : $\mathrm{div}\,\vec{\Gamma} = -4\pi G\rho(r)$

Pour $R_1 < r < R_T$: $-\dfrac{1}{r^2}\dfrac{dr^2\Gamma_0}{dr} = -4\pi G\rho(r)$

$\dfrac{dr^2\Gamma_0}{dr} = 4\pi G\rho(r) \cdot r^2$

$2r\Gamma_0 + r^2 \cdot 0 = 4\pi G\rho(r) r^2$

$\rightarrow \boxed{\rho(r) = \dfrac{\Gamma_0}{2\pi r G}}$ soit $\rho(r) = \dfrac{M}{2\pi r R_T^2}$; $\rho' = \dfrac{R_T}{R_1}\rho = \dfrac{3M}{4\pi R_T^2 R_1}$

$\rho(R_1) = \dfrac{M}{2\pi R_T^2 R_1}$; $\rho' = \dfrac{3}{2}\rho(R_1)$