

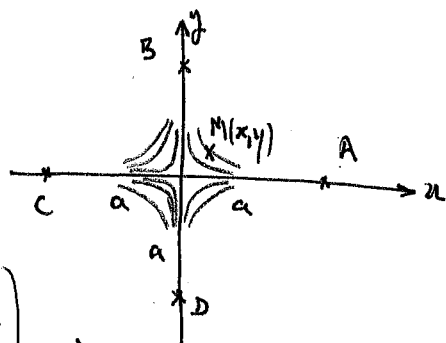
EM112. Quatre charges aux sommets d'un carré

1. Superposition: $V(M) = V_A(M) + V_B(M) + V_C(M) + V_D(M)$

$$V = \frac{q}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{(a-x)^2 + y^2}} + \frac{-1}{\sqrt{x^2 + (a-y)^2}} + \frac{1}{\sqrt{(a+x)^2 + y^2}} + \frac{-1}{\sqrt{x^2 + (a+y)^2}} \right)$$

$$= \frac{q}{4\pi\epsilon_0 a} \left(\frac{1}{\sqrt{(1-\frac{x}{a})^2 + \frac{y^2}{a^2}}} - \frac{1}{\sqrt{\frac{x^2}{a^2} + (1-\frac{y}{a})^2}} + \frac{1}{\sqrt{(1+\frac{x}{a})^2 + \frac{y^2}{a^2}}} - \frac{1}{\sqrt{\frac{x^2}{a^2} + (1+\frac{y}{a})^2}} \right)$$

$$\frac{1}{\sqrt{(1-\frac{x}{a})^2 + \frac{y^2}{a^2}}} = \frac{1}{\sqrt{1 - \frac{2x}{a} + \frac{x^2+y^2}{a^2}}} \approx \frac{1}{\sqrt{1 - \frac{2x}{a} + \frac{x^2+y^2}{a^2}}} \approx 1 + \frac{x}{a} - \frac{x^2+y^2}{2a^2} + \frac{3}{4} \frac{x^2}{a^2}$$



$$(1+x)^2 \approx 1 + 2x + \frac{1}{2}x^2$$

$$V \approx V_0 \left[\left(1 + \frac{x}{a} - \frac{x^2+y^2}{2a^2} \right) - \left(1 + \frac{y}{a} - \frac{x^2+y^2}{2a^2} \right) + \left(1 - \frac{x}{a} - \frac{x^2+y^2}{2a^2} \right) - \left(1 - \frac{y}{a} - \frac{x^2+y^2}{2a^2} \right) + \frac{3}{4} \left(\frac{x^2}{a^2} + \frac{y^2}{a^2} \right) \right]$$

$$V \approx V_0 \cdot \frac{3}{4} \left(\frac{x^2}{a^2} + \frac{y^2}{a^2} \right) \rightarrow V = \frac{3q}{4\pi\epsilon_0 a^3} (x^2 - y^2)$$

$$2. \vec{E} = - \frac{\partial V}{\partial x} \vec{e}_x - \frac{\partial V}{\partial y} \vec{e}_y = - \frac{3q}{4\pi\epsilon_0 a^3} (x \vec{e}_x - y \vec{e}_y) = \vec{E}$$

$$\frac{dx}{E_x} = \frac{dy}{E_y} \rightarrow \frac{dx}{x} = - \frac{dy}{y}$$

$$\hookrightarrow xy = \text{cte} \quad \text{hyperboles}$$