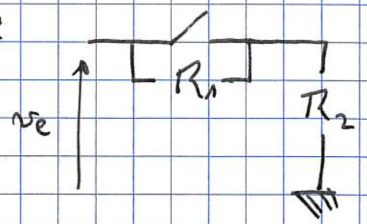
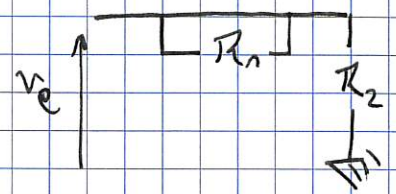
EL 44 _ Filtre du 1^{er} ordre

1 a _ BF : schéma équivalent :



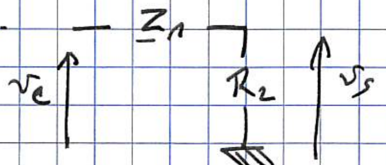
$$v_s = \frac{R_2}{R_1+R_2} v_e$$

HF :



$$v_s = v_e$$

b.



Pont diviseur de tension : $\frac{\underline{v_s}}{\underline{v_e}} = \frac{R_2}{R_2 + \underline{Z}_1}$

où $\underline{Z}_1 = \frac{R_1/jC\omega}{R_1 + \frac{1}{jC\omega}} = \frac{R_1}{1+jR_1C\omega}$

$$\Rightarrow \underline{H} = \frac{R_2}{R_2 + \frac{R_1}{1+jR_1C\omega}} = R_2 \frac{1+jx}{R_2(1+jx)+R_1} \quad \text{où } x = R_1C\omega$$

$$= \frac{R_2}{R_1+R_2}\,\frac{1+jx}{1+jR_2x/(R_1+R_2)} \Rightarrow \alpha = \frac{R_2}{R_1+R_2} = 1{,}0\,10^{-2}$$

$$\tau = \alpha R_1 C = 4{,}7\,10^{-6}\,s$$

D'où $$\boxed{\underline{H} = \alpha \frac{1+j\frac{\tau}{\alpha}\omega}{1+j\tau\omega}}$$

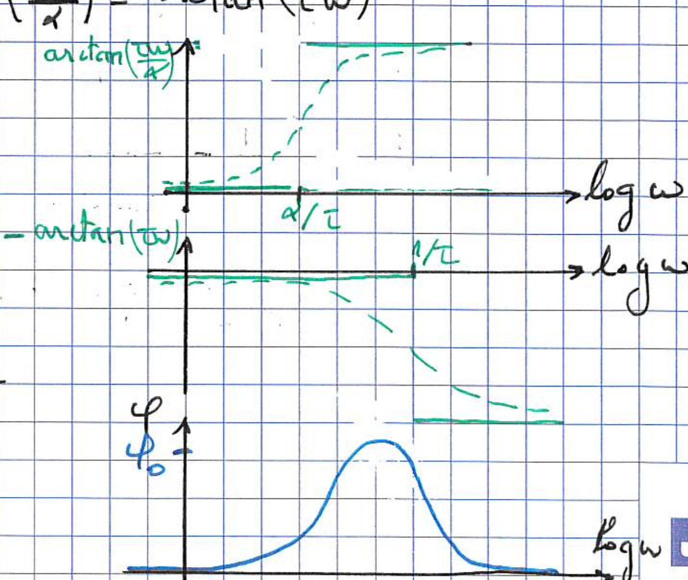
c _ BF : $\omega\tau \ll 1$: $\underline{H} \simeq \alpha$:)

HF : $\omega\tau \gg 1$: $\underline{H} \simeq 1$:)

2 _ a _ $\varphi = \arg(\underline{H}) = \arctan\left(\frac{\tau\omega}{\alpha}\right) - \arctan(\tau\omega)$

BF : $\varphi \simeq 0$

HF : $\varphi \simeq 0$



Donc graphiquement, φ passe par un maximum.

$$\frac{d\varphi}{d\omega} = \frac{\frac{\tau}{\alpha}}{1+\left(\frac{\tau\omega}{\alpha}\right)^2} - \frac{\tau}{1+(\tau\omega)^2} = \frac{\frac{\tau}{\alpha}\left(1+(\tau\omega)^2\right) - \tau\left(1+\left(\frac{\tau\omega}{\alpha}\right)^2\right)}{\left(1+\left(\frac{\tau\omega}{\alpha}\right)^2\right)\left(1+(\tau\omega)^2\right)}$$

Numérateur : $\frac{\tau}{\alpha} - \tau + \left(\frac{\tau}{\alpha} - \frac{\tau}{\alpha^2}\right)(\tau\omega)^2 = \left(\frac{\tau}{\alpha} - \tau\right)\left(1 - \frac{1}{\alpha}(\tau\omega)^2\right)$

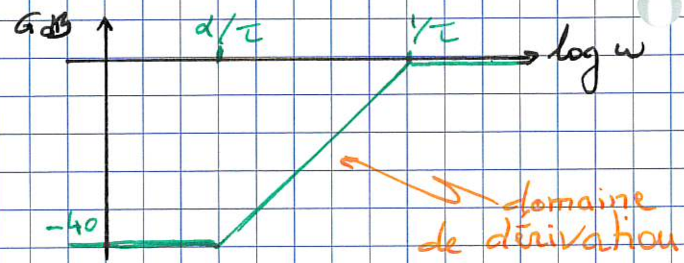
s'annule pour $\boxed{\omega = \frac{\sqrt{\alpha}}{\tau}}$ $\omega = 2,1\ 10^4\ \text{rad s}^{-1}$

$\boxed{\varphi_0 = \varphi\left(\frac{\sqrt{\alpha}}{\tau}\right) = \arctan\left(\frac{1}{\sqrt{\alpha}}\right) - \arctan(\sqrt{\alpha})}$ $\underline{\varphi_0 = 1,37\ \text{rad}}$

b - * $\omega \ll \frac{\alpha}{\tau} \ll \frac{1}{\tau}$: $\underline{H} \simeq \alpha \Rightarrow G_{dB} \simeq 20 \log \alpha$

* $\frac{\alpha}{\tau} \ll \omega \ll \frac{1}{\tau}$: $\underline{H} \simeq j\tau\omega \Rightarrow G_{dB} \simeq 20\log\tau + 20\log\omega$

* $\omega \gg \frac{1}{\tau} \gg \frac{\alpha}{\tau}$: $\underline{H} \simeq 1 \rightarrow G_{dB} \simeq 0$

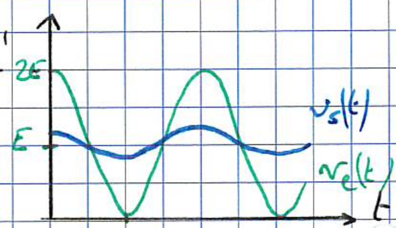


$\omega_1 = \frac{\alpha}{\tau} = 2,1\ 10^3\ \text{rad s}^{-1} \longrightarrow f_1 = 3,4\ 10^2\ \text{Hz}$

$\omega_2 = \frac{1}{\tau} = 2,1\ 10^5\ \text{rad s}^{-1} \longrightarrow f_2 = 34\ \text{kHz}$

3 a- Couplage DC pour visualiser l'offset.

b- $\omega \ll \omega_1$: signal très atténué non déformé



c - $\omega_1 \ll \omega \ll \omega_2$: signal intégré et atténué