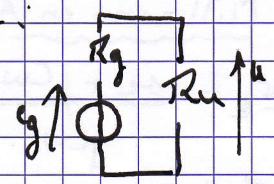
EL32_ Adaptateurs d'impédance à composants réactifs

NB : Cas simple d'adaptation d'impédance :



$$P_u = \frac{u^2}{R_u} = e_g \frac{R_u}{(R_u+R_g)^2}$$

$$\frac{dP_u}{dR_u} = e_g \frac{(R_u+R_g)^2 - 2R_u(R_u+R_g)}{(R_u+R_g)^4}$$

$$= e_g \frac{-R_u^2+R_g^2}{(R_u+R_g)^4} = 0 \quad \text{si} \quad \underline{R_u = R_g}$$

Structure a : $Z_a = Z_C + \frac{Z_L R_u}{Z_L + R_u} = \frac{1}{jC\omega} + \frac{jL\omega R_u}{jL\omega + R_u}$

$$\to Z_a = \frac{-j}{C\omega} + \frac{jL\omega R_u (R_u - jL\omega)}{R_u^2 + (L\omega)^2} = \frac{(L\omega)^2 R_u}{R_u^2+(L\omega)^2} + j\left(\frac{R_u^2 L\omega}{R_u^2+(L\omega)^2} - \frac{1}{C\omega}\right)$$

Adaptation d'impédance $Z_a = R_g \Rightarrow \left| \begin{array}{ll} R_g = R_u \dfrac{(L\omega)^2}{R_u^2+(L\omega)^2} & (a) \\ \dfrac{R_u^2 L\omega}{R_u^2+(L\omega)^2} = \dfrac{1}{C\omega} & (b) \end{array} \right.$

(a) : $\Rightarrow \underline{R_u > R_g}$

$R_u (a) - L\omega (b)$: $R_u R_g = \frac{L\omega}{C\omega} \Rightarrow \frac{R_u R_g}{L\omega} = \frac{1}{C\omega}$ (c)

Dans (b) : $\frac{R_u^2 L\omega}{R_u^2 + (L\omega)^2} = \frac{R_u R_g}{L\omega} \to (L\omega)^2 = \frac{R_u^2 R_g}{R_u - R_g} \to \boxed{L = \frac{R_u}{\omega}\sqrt{\frac{R_g}{R_u - R_g}}}$

Avec (c) $C = \frac{L}{R_u R_g} \to \boxed{C = \frac{1}{R_g \omega}\sqrt{\frac{R_g}{R_u - R_g}}}$

Structure b : $Z_b = Z_L // (Z_C + R_u) = \frac{jL\omega\left(R_u + \frac{1}{jC\omega}\right)}{jL\omega + R_u + \frac{1}{jC\omega}} = \frac{\frac{L}{C} + jLR_u\omega}{R_u + j\left(L\omega - \frac{1}{C\omega}\right)}$

$$Z_b = \frac{\left(\frac{L}{C} + jLR_u\omega\right)\left(R_u - j\left(L\omega - \frac{1}{C\omega}\right)\right)}{R_u^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}$$

$$= \frac{\frac{LR_u}{C} + L\omega R_u(\;)}{R_u^2 + \left(L\omega - \frac{1}{C\omega}\right)^2} + j\frac{LR_u^2\omega - \frac{L}{C}(\;)}{R_u^2 + (\;)^2} = R_g$$: adaptation d'impédance

$$\Rightarrow \left| \begin{array}{l} \dfrac{\frac{LR_u}{C} + LR_u\omega(\;)}{R_u^2 + (\;)^2} = R_g \\ LR_u^2\omega - \dfrac{L}{C}(\;) = 0 \end{array} \right. \Rightarrow \left| \begin{array}{ll} \dfrac{L}{C}R_u + LR_u\omega(\;) = R_g\left(R_u^2 + (\;)^2\right) & (a) \\ \dfrac{L}{C}(\;) = LR_u^2\omega & (b) \end{array} \right.$$

$(a).(\;) - (b)R_u$: $LR_u\omega(\;)^2 = R_g\left(R_u^2 + (\;)^2\right).(\;) - LR_u^3\omega$

$$LR_u\omega\cancel{\left(R_u^2 + (\;)^2\right)} = R_g(\;)\cancel{\left(R_u^2 + (\;)^2\right)}$$

$$L\omega R_u = R_g\left(L\omega - \frac{1}{C\omega}\right) \Rightarrow L\omega(R_u - R_g) = -\frac{R_g}{C\omega}$$

$\rightarrow \underline{R_u < R_g}$; $\boxed{L\omega = \frac{R_g}{R_g - R_u} \frac{1}{C\omega}}$

$$L\omega - \frac{1}{C\omega} = \left(\frac{R_g}{R_g - R_u} - 1\right)\frac{1}{C\omega} \rightarrow L\omega - \frac{1}{C\omega} = \frac{1}{C\omega}\,\frac{R_u}{R_g - R_u}$$

Dans (b) : $\frac{L}{C^2\omega}\,\frac{R_u}{R_g - R_u} = L R_u^2 \omega \Rightarrow \boxed{C = \frac{1}{\omega}\sqrt{\frac{1}{R_u(R_g - R_u)}}}$