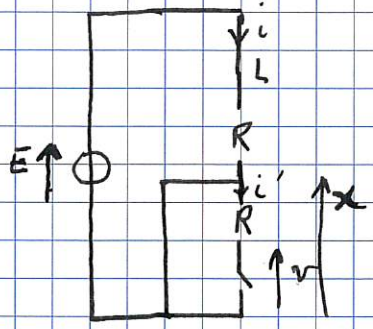
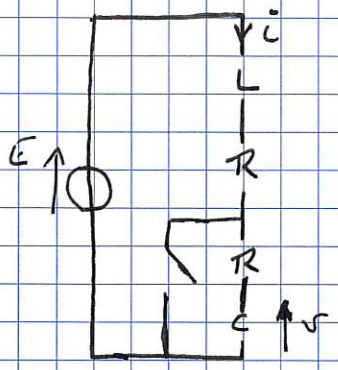
EC24 - Circuit oscillant

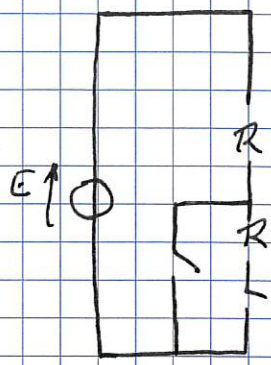
1 - $t<0$ (RP)



$t=0^+$



$t \to \infty$ (RP)



$i = \frac{E}{R}$

$x = 0 = v + Ri' = v$

Continuité de i de v

	0^-	0^+	∞
i	E/R	E/R	0
v	0	0	E
x	0	$-E$	E

2 - Loi des mailles :

$$E = L\frac{di}{dt} + 2Ri + v \quad \text{où} \quad i = C\dot{v}$$

$$\rightarrow E = LC\ddot{v} + 2RC\dot{v} + v \Rightarrow \boxed{\ddot{v} + \frac{2R}{L}\dot{v} + \frac{1}{LC}v = \frac{E}{LC}}$$

Discriminant du polynôme caractéristique : $\Delta = \frac{4R^2}{L^2} - \frac{4}{LC}$

Pour le régime oscillant (pseudopériodique)

$$r_\pm = \frac{R}{L} \pm j\sqrt{\frac{|\Delta|}{4}} = \frac{R}{L} \pm j\underbrace{\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}_{\Omega}$$

$$v = e^{-Rt/L}\left(A\cos(\Omega t) + B\sin(\Omega t)\right)$$

C.I : $v(0^+) = 0 \Rightarrow A = 0 \Rightarrow v(t) = Be^{-Rt/L}\sin(\Omega t)$

$$\dot{v}(t) = Be^{-Rt/L}\left[-\frac{R}{L}\sin(\Omega t) + \Omega\cos(\Omega t)\right]$$

$$\dot{v}(0^+) = \frac{i(0^+)}{C} = \frac{E}{RC} = B\Omega \Rightarrow B = \frac{E}{\Omega RC}$$

$$\rightarrow \boxed{v(t) = \frac{E}{\Omega RC}e^{-Rt/L}\sin(\Omega t)}$$